

## Zadanie 5.1

Obliczyć pochodne funkcji

1.  $f(x) = e^7 + \arctan x - 3 \ln 3$

2.  $f(x) = \arcsin x - \sqrt[3]{x}$

3.  $f(x) = \sqrt{x} - \frac{5}{7} \sqrt[5]{x^3} - 2\sqrt{x^5}$

4.  $f(x) = \frac{1}{\sqrt{x}} + \frac{2\sqrt{x}}{\sqrt[3]{x} \cdot \sqrt{x^3}}$

5.  $f(x) = 2x^{\frac{5}{2}} - 3x^{\frac{1}{6}} + 7x^{\frac{4}{7}}$

6.  $y = \frac{\sqrt{x}}{x^2 + \ln \cos x}$

7.  $y = \frac{x^5 \operatorname{tg} x}{\arcsin x}$

8.  $f(x) = \frac{\sin(x^2) + \arctan x}{\ln x \operatorname{tg} x}$

9.  $f(x) = \sin(\ln(\arcsin(e^x)))$

10.  $f(x) = (x^3 + 2x^2 - x + 4)^{\cos x}$

11.  $f(x) = 3^{\cos x} \tan(x^2 + \sqrt{x})$

12.  $f(x) = \frac{\arctan x - x}{e^{x^3+x}}$

13.  $f(x) = 2^{\sin x} \ln(x^2 + \sqrt{x})$

14.  $f(x) = \frac{\arcsin x + x}{e^{x^2+1}}$

15.  $f(x) = (\tan x)^{\sqrt[3]{x+2}}$

16.  $f(x) = (\tan x)^{\sin x+x}$

17.  $f(x) = \frac{e^x+3}{\arcsin x^2}$

18.  $f(x) = \arcsin(x^2 + 3) \cdot e^{\sqrt{x} + \frac{1}{\sqrt{x}}}$

19.  $f(x) = \ln(x^2 + \sqrt{x}) \cdot \arctan(x + \frac{1}{\sqrt[3]{x}})$

20.  $f(x) = \frac{\arctan x + x^3}{\ln x^2}$

21.  $f(x) = (\cos x)^{\tan x+x}$

22.  $h(x) = (\ln x)^{\sqrt[3]{x+2}}$

23.  $f(x) = \frac{\sin x \cdot \ln x}{\arcsin x}$

24.  $f(x) = \frac{x \cdot e^x}{\sqrt{2x}}$

25.  $f(x) = \ln(\sqrt{\arcsin(e^{2x})})$

## Zadanie 5.2

Obliczyć granice funkcji

1.  $\lim_{x \rightarrow 1} \frac{x^5 - 2x^3 + 4x - 3}{x^5 - 1}$
2.  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin x}$
3.  $\lim_{x \rightarrow \infty} \frac{\ln(\ln x)}{x}$
4.  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\cos x \sin x}$
5.  $\lim_{x \rightarrow 0^+} \frac{e^x - e^{-x} - 2x}{x - \sin x}$
6.  $\lim_{x \rightarrow \infty} x(1 - e^{\frac{1}{x}})$
7.  $\lim_{x \rightarrow 0} \frac{1}{\sin x} (e^{x^2} - 1)$
8.  $\lim_{x \rightarrow 0} \frac{2 \ln \cos x + x^2}{x^2 + 2 \cos x - 2x}$
9.  $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{\sin x} \right)$
10.  $\lim_{x \rightarrow 1^+} (1 - x) \ln(1 - x)$
11.  $\lim_{x \rightarrow 0^+} (\sin x)^{\sin x}$
12.  $\lim_{x \rightarrow 1} \left( \frac{3}{x^3 - 1} - \frac{4}{x^4 - 1} \right)$
13.  $\lim_{x \rightarrow \infty} \ln x (\pi - 2 \arctan x)$
14.  $\lim_{x \rightarrow 0^+} \sqrt{x} \ln^2 x$
15.  $\lim_{x \rightarrow 1^+} \frac{\ln x}{\sqrt{x^2 - 1}}$
16.  $\lim_{x \rightarrow \infty} (x - \ln x)$
17.  $\lim_{x \rightarrow \infty} (e^x - x^3)$
18.  $\lim_{x \rightarrow 0^+} \left( \ln \frac{1}{x} \right)^x$
19.  $\lim_{x \rightarrow 0} \left( \operatorname{tg} \frac{x}{2} \right)^{\frac{1}{\ln x}}$
20.  $\lim_{x \rightarrow \infty} \frac{x^2 + 4x - 5}{2x - 4} \sqrt{x^5 - 4x^3 + 2x^2 - 7}$

## Zadanie 5.3

Wyznaczyć asymptoty funkcji

1.  $f(x) = \frac{x+2}{x-3}$
2.  $f(x) = \frac{x^3}{(x-1)^2}$
3.  $f(x) = \frac{x^2}{x^4-4}$

4.  $f(x) = \frac{(x+1)^2}{2x}$
5.  $f(x) = x\sqrt{\frac{x}{2-x}}$
6.  $f(x) = \frac{x^2-x}{(x+1)(x+3)}$
7.  $f(x) = \frac{\sin x}{x}$
8.  $f(x) = x - \frac{4}{x^2}$
9.  $f(x) = \frac{x^2-3}{x-2}$
10.  $f(x) = \frac{x}{\ln x}$

### Zadanie 5.4

Wyznaczyć ekstrema lokalne i przedziały monotoniczności funkcji

1.  $y = 3x^4 + 8x^3 - 6x^2 - 24x$
2.  $y = -2x^4 + 24x^3 - 92x^2 + 120x + 2$
3.  $y = x^3 + 12x^2 + 36x - 50$
4.  $y = x(3-x)^2$
5.  $y = x^4 - 8x^3 + 22x^2 - 24x + 12$
6.  $y = -x - \frac{1}{x}$
7.  $y = \frac{x}{1+x^2}$
8.  $y = \frac{x^2}{x^2-1}$
9.  $y = \frac{x}{2} + \frac{2}{x}$
10.  $y = \frac{x^4}{x^3-1}$
11.  $y = \frac{x^2+x+1}{x^2-x+1}$
12.  $y = \frac{e^x}{x-3}$
13.  $y = \frac{e^{x^2}}{x^2-2}$
14.  $y = \frac{e^x+2}{x^2-13}$
15.  $y = \frac{\ln x}{x}$
16.  $y = \frac{x}{\ln x}$
17.  $y = x^2 \ln x$
18.  $y = e^{-x}x^3$
19.  $y = \frac{x^2}{2} \arctan(x+1) - \frac{1}{2}x + \frac{1}{2} \ln(x^2 + 2x + 2)$
20.  $y = (x^2 - 2x) \ln x - \frac{3}{2}x^2 + 4x$

## Zadanie 5.5

Zbadać przebieg zmienności funkcji

1.  $f(x) = \frac{x+1}{x-3}$

2.  $f(x) = \frac{2x+7}{x+1}$

3.  $f(x) = x\sqrt{4-x^2}$

4.  $y = \frac{\ln x}{x}$

5.  $y = \frac{x}{\ln x}$

6.  $y = \frac{\ln x}{x}$

7.  $y = x \ln x$

8.  $y = e^{-x}x^2$

9.  $y = \frac{2x-3}{x+1}$

10.  $y = \frac{x}{1+x^2}$

11.  $y = \frac{x^2}{x^2-4}$

12.  $f(x) = x\sqrt{4-x^2}$

13.  $\frac{1}{x^2+1}$

14.  $\frac{x^3}{x^2-1}$

## Zadanie 5.6

Uzasadnić, że

1.  $\arctan \frac{1}{x} + \arctan x = \frac{\pi}{2}, x > 0$

2.  $2x \arctan x \geq \ln(1+x^2), x \in \mathbb{R}$